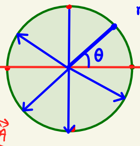
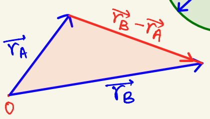
① motion plane ; distance of object is always constant from a fixed point.

② Radius vector. radius vector has fixed magnitude but variable direction.

direction — from center of circle to position of object.





Frequency

$t \longrightarrow n$

$1 \longrightarrow \frac{n}{t} = \frac{1}{sec} = sec^{-1} =$ Hertz. (Hz)

revolution per second. (rps)

revolution per minute. (rpm)

1 rpm = 60 rps.

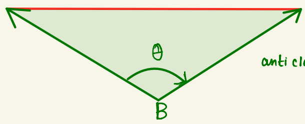
Angular Displacement —

change in angular position. $\theta = (\theta_2 - \theta_1)$

angle through which radius vector of particle rotate in given time interval

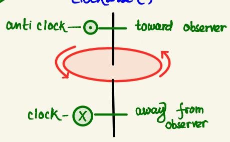
angular displacement $(\Delta\theta) = \frac{arc\ (\Delta s)}{radius\ (r)}$ (radian)

Ⓐ



① axial vector.

② anticlockwise (+) clockwise (−)



Time period

time for complete one revolution.

let n be the frequency.

$n \longrightarrow 1$

$1 \longrightarrow \frac{1}{n} = T$

$$\text{Time period} = \frac{1}{\text{frequency}}$$

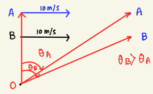
Anugular Velocity ω

$$\omega_{avr} = \frac{\text{Total angular displacement } \Delta\theta \text{ (rad)}}{\text{total time } \Delta t \text{ (sec)}}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$ dimension $= T^{-1}$

$\omega = 2$ rps $1 \longrightarrow 2\pi$

$\omega = 4\pi$ rad/sec. $2 \longrightarrow 4\pi$



$\theta_B > \theta_A$

angle $= \frac{arc = const}{radius}$

angle $\propto \frac{1}{radius}$

① radius = 2m
$\theta = (t^2 - 4)$ rad.

ⓐ θ in first 2 sec (4 rad)
ⓑ ω_{av} t = 4 sec 4 rad/sec
ⓒ ω_{in} t = 5 sec 10 rad/sec
ⓓ α_{in} t = 10 sec. 2 rad/sec^2

$\theta = t^2 - 4$
$\theta_2 = 4 - 4 = 0$
$\theta_0 = -4$
$\theta = \theta_2 - \theta_0$
$= 4$ unit

ⓑ $\omega_{av} = \frac{\Delta\theta}{\Delta t}$
$= \frac{\theta_4 - \theta_0}{4}$
$= \frac{12 - (-4)}{4}$
$= 4$ rad/sec

ⓒ $\theta = t^2 - 4$
$\omega = \frac{d\theta}{dt}$
$= 2t$ $\alpha = \frac{d\omega}{dt} = 2$
$\omega_5 = 2 \cdot 5$
$= 10$ rad/sec

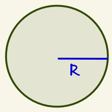
Angular accelaration

linear accelaration $= a = \frac{dv}{dt}$ $\alpha = \frac{d\omega}{dt}$

$v = v(x)$

$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$

$a = v\frac{dv}{dx}$ $\alpha = \omega\frac{d\omega}{d\theta}$



linear velocity = radius of circular path X angular velocity

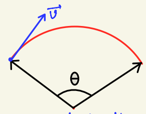
$v = R\omega$

$\frac{d}{dt}(v) = \frac{d}{dt}(R\omega)$ unit $\alpha =$ rad/sec^2

$a = R\frac{d\omega}{dt}$

$a = R\alpha$

linear accelaration = radius of circular path X angular accelaration



distance covered in time dt is $ds = vdt$

angular displacement $= \frac{ds}{R} = \frac{vdt}{R} = d\theta$

$d\theta = \frac{vdt}{R}$

$\frac{d\theta}{dt} = \frac{v}{R}$

$\omega = \frac{v}{R}$ $v = \omega R$

Linear velocity= angular velocity x radius of circular path

Non uniform acceln $\alpha = 4t^3 + 2t$ rad/sec^2.

Find avarage angular speed of body at t=2 sec. if it start from rest.

$a = Kt$

$\frac{dv}{dt} = Kt$

$\int dv = \int K t\, dt$

$\frac{d\omega}{dt} = 4t^3 + 2t$

$\int_0^{\omega} d\omega = \int_0^{t} (4t^3 + 2t)\, dt$

$\omega = \left[t^4 + t^2\right]_0^2$

$= 16 + 4 = 20$ rad/sec

Equation of motion under constant acceleration

① $v = u + at$ $\omega = \omega_0 + \alpha t$
② $s = ut + \frac{1}{2}at^2$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
③ $v^2 = u^2 + 2as$ $\omega^2 = \omega_0^2 + 2\alpha\theta$
④ $S_n = u + \frac{1}{2}a(2n-1)$ $\theta_n = \omega_0 + \frac{1}{2}\alpha(2n-1)$

For the case of circular motion,
$v \sim \omega$
$a \sim \alpha$
$s \sim \theta$

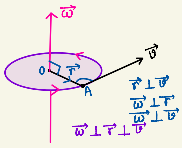
In the case of non uniform angular acceleration

$\alpha = \frac{d\omega}{dt}$ $\omega = \frac{d\theta}{dt}$

$\alpha = \omega\frac{d\omega}{d\theta}$

$v = \omega r$

$\vec{v} = \vec{\omega} \times \vec{r}$



$\vec{r} \perp \vec{v}$
$\vec{\omega} \perp \vec{r}$
$\vec{\omega} \perp \vec{v}$
$\vec{\omega} \perp \vec{r} \perp \vec{v}$

A solid body rotates about a stationary axis with an angular retardation $\alpha = K\sqrt{\omega}$ Where ω Is the angular velocity of the body.find the time after which body will come to rest if at t=0 angular velocity of body was ω_0

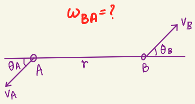
A particle start moving in a circular path of radius 2m with initial speed 2m/s at constant angular acceleration .after two complete revolution, its speed becomes 8m/s. Find the angular acceleration of the particle.

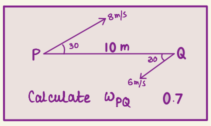
A wheel is subjected to uniform angular acceleration about its axis .initially it was at rest. In first two second it rotates through angle θ_1. In next two second it rotates through θ_2. Find the ratio of θ_2/θ_1 .

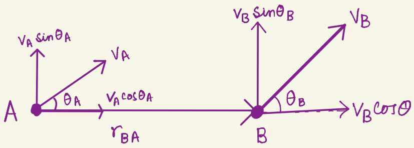
The length of second hand in a watch is 1 cm . Find the magnitude of change in velocity of its tip in 15 sec. find also the magnitude of Avarage acceleration.

$\frac{\pi}{30}$

$\omega_{BA} = ?$







Angular velocity of B with respect to A?

$\frac{V_B \sin\theta}{r_{BA}}$ $\frac{V_B\sin\theta_B - V_A\sin\theta_A}{r_{BA}}$

Whenever particle perform circular motion then linear velocity is perpendicular to radius vector

Is it necessary for a body to perform circular motion for having angular velocity?

$$\omega_{BA} = \frac{\text{Perpendicular component of } V_{BA}}{\text{Distance}}$$

The length of second hand in a watch is 1 cm . Find the magnitude of change in velocity of its tip in 15 sec. find also the magnitude of Avarage acceleration.

A solid body rotates about a stationary axis with an angular retardants ~~ai~~. $\alpha = K\sqrt{\omega}$ Where ω Is the angular velocity of the body. find the time after which body will come to rest if at t=0 angular velocity of body was ω_0

angular retardation $\alpha = -K\sqrt{\omega}$

$$\frac{d\omega}{dt} = -K\sqrt{\omega} \Rightarrow \int_{\omega_0}^{0} \frac{d\omega}{\sqrt{\omega}} = -\int_0^t K\,dt$$

$$\Rightarrow -2\sqrt{\omega_0} = -Kt$$

$$t = \frac{2\sqrt{\omega_0}}{K}$$

A wheel is subjected to uniform angular acceleration about its axis .initially it was at rest. In first two second it rotates through angle θ_1. In next two second it rotates through. θ_2. Find the ratio of θ_2/θ_1 .

$$\theta = \frac{1}{2}\alpha t^2$$

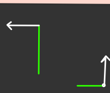
$$\theta_1 = 2\alpha$$

$$\theta_2 + \theta_1 = \frac{1}{2}\alpha 4^2 = 8\alpha$$

$$\theta_2 = 6\alpha$$

$$\therefore \frac{\theta_2}{\theta_1} = \cancel{6\alpha} = 3$$

The length of second ha
the magnitude of chang
sec. find also the magni



A particle start moving in a circular path of radius 2m with initial speed 2m/s at constant angular acceleration .after two complete revolution, its speed becomes 8m/s. Find the angular acceleration of the particle.

Tangential accelaration

. Acting always along the velocity

Increase the speed.

Acting perpendicular to radius vector.

Acting perpendicular to angular velocity vector.

Vector form $\vec{a}_T = \vec{\alpha} \times \vec{r}$

Magnitude = radius of circular path X angular acceleration

23. $m = 1\,kg$
$u = 50\,m/s$
$\theta = 60$
$F = 0.2v$ $\boxed{a = 0.2v}$
$t = 10$
$s = ?$ $s = ut - \frac{1}{2}at^2$

$a = v\frac{dv}{dx}$

$0.2\cancel{v} = \cancel{v}\frac{dv}{dx}$

$0.2\,dx = \int dv$

$v = v(x)$

$v = ax$

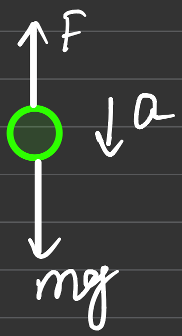
$\frac{dx}{dt} = ax$

$\int_0^x \frac{dx}{x} = a\int_0^{10} dt$

$\frac{1}{2} \times \cancel{2}^{5} \times 0.25 = \frac{1\cancel{25}^{5}}{\cancel{200}_{8}}$

$\frac{0.25}{400} =$

$= 0.\ \ \ 0.625$



$mg - F = ma$

$T = m_2 a$ —(i)

$m_1 g - T = m_1 a$ —(ii)

$m_1 g = (m_1 + m_2)a$

$a = \frac{m_1 g}{m_1 + m_2}$

$s = \frac{1}{2}at^2$

$a = \frac{10^4}{10 \times 1000}$
$= 1$

$F = 4 \times 10^3 \times 1$

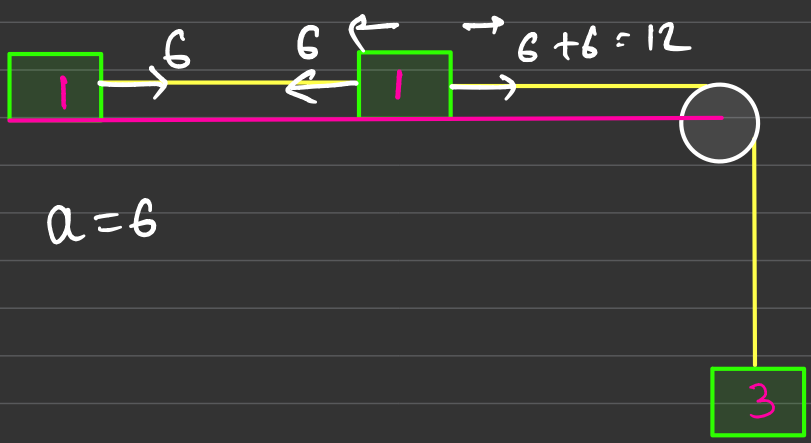
$a = 6$

$\boxed{F = m(g-a)}$

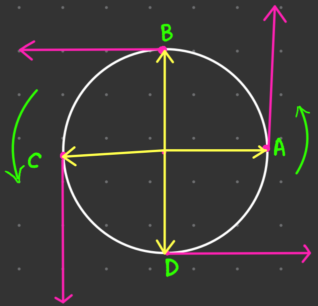
$F = (m-x)(g+a)$

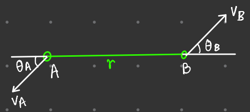
$m(g-a) = (m-x)(g+a)$

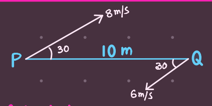
$x = \frac{2ma}{(g+a)}$



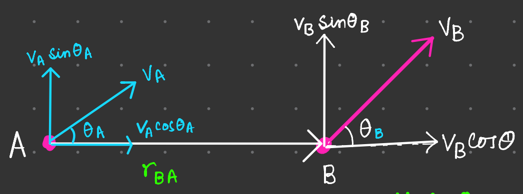
$a = 6$







Calculate ω_{PQ} 0.7



Angular velocity of B with respect to A?

$$\frac{V_B \sin\theta}{r_{BA}}$$

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$$\omega_{BA} = \frac{\text{Perpendicular component of } V_{BA}}{\text{Distance}}$$

Is it necessary for a body to perform circular motion for having angular velocity?

$\vec{v} = \vec{\omega} \times \vec{r}$

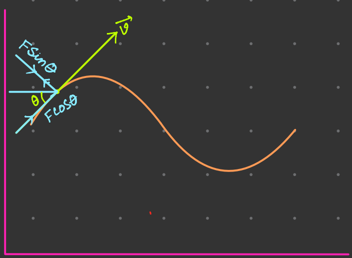
$\frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}$

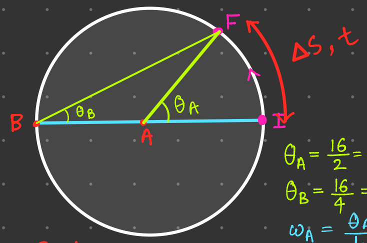
$\vec{a} = (\vec{\omega} \times \vec{v}) + (\vec{\alpha} \times \vec{r})$

$\vec{a} = \vec{a_c} \times \vec{a_T}$

$\vec{a_T}$ = Tangential accelaration

$\vec{a_c}$ = centripital accelaration





$\theta_A = \frac{16}{2} = 8$ rad

$\theta_B = \frac{16}{4} = 4$ rad

$\omega_A = \frac{\theta_A}{t} =$

$\omega_B = \frac{\theta_B}{t}$

Radius = R = 2 m

$\Delta S = 16$ m

t = 4 sec

Speed = const

F

angle between velocity and accelaration $\theta = 0^\circ$

$\theta = 0^\circ$ $v(\uparrow)$

$\theta = 180^\circ$ $v(\downarrow)$ (1-D motion)

when $\theta \neq 0, 180^\circ$ (2-D motion)

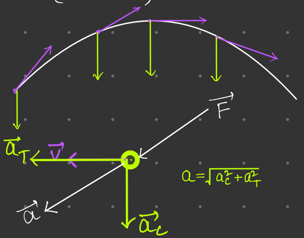
a_T = Tangential accelaration $= \frac{F\cos\theta}{m} =$

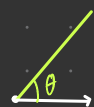
a_c = centripital accelaration $= \frac{F\sin\theta}{m}$

Total accelaration $= a = \sqrt{a_T^2 + a_c^2}$

$= \frac{F}{m}\sqrt{\cos^2\theta + \sin^2\theta}$

$\boxed{a = \frac{F}{m}}$





u, a = tangential accelaration

$\theta = 0^\circ$

Tangential accelaration

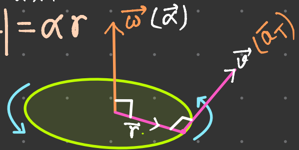
Rate of change of speed or magnitude of velocity

Act parallel to velocity

If a particle moving in a circular path with constant speed then tangential acceleration is zero

$\vec{a} = \vec{\alpha} \times \vec{r}$

$|\vec{a_T}| = \alpha r$



$\vec{r} \perp \vec{v} \perp \vec{\omega}$

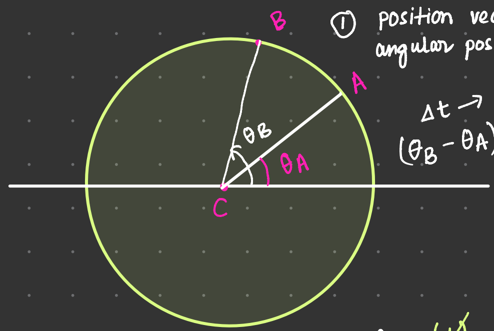
Freequency (Hertz)

60 sec ⟶ 1

1 sec ⟶ $\frac{1}{60 \text{ sec}}$ = sec^{-1} = Hertz

Revolution per minute ① second hand of clock

Revolution per second $\frac{1}{60}$



① position vector

angular position

$\Delta t \rightarrow (A \rightarrow B)$

$(\theta_B - \theta_A) = \Delta\theta$

Timeperiod (T)

T ⟶ 1

1 ⟶ $\boxed{\frac{1}{T} = f}$

Angular displacement $= \frac{\text{Arc}(L)}{\text{Radius}(L)}$ = radian

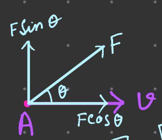
dimensionless.

Angular displacement circular motion must?

$\boxed{\text{Vector}}$

$\Delta\theta = \frac{\Delta S}{r}$

$\Delta S = \Delta\theta \cdot r$



$\boxed{\theta = 0, 180}$

$\boxed{\vec{v}@ = 0^\circ}$

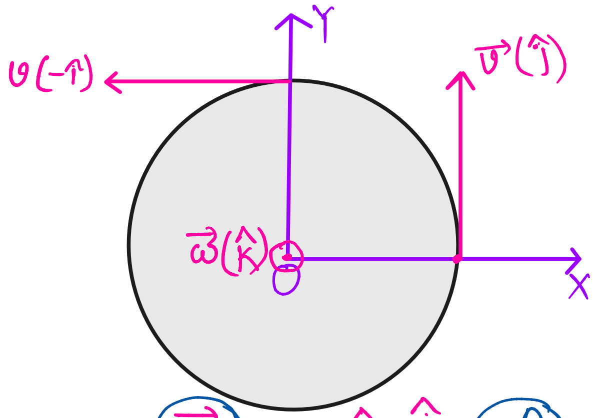
$\boxed{\theta = 0, 180}$

Centripetal acceleration

$$\vec{a}_{cp} = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$a_{cp} = |\vec{a}_{cp}| = \omega v = \omega^2 R = v^2/R$$

Centripetal acceleration is always acting perpendicular to velocity . Hence this is responsible for change in direction of velocity.



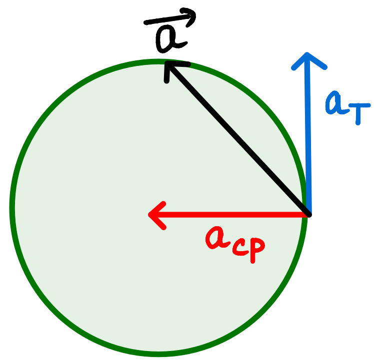
$$\vec{a}_{cp} = \hat{k} \times \hat{j} = -\hat{i}$$

Net acceleration

$$\vec{a} = \vec{a}_T + \vec{a}_{cp}$$

$$a = |\vec{a}| = \sqrt{a_T^2 + a_{cp}^2} = \sqrt{(\alpha r)^2 + \left(\frac{v^2}{r}\right)^2}$$

direction of accelaration $\theta = ?$



Numerical

1. A particle is moving with constant angular acceleration of 4 rad/s^2 on circular path. At the time t=0 particle was at rest then find the time at which magnitude of a_T and a_{cp} will be equal. (t=1/2sec) $\alpha = 4$

$$a_T = a_{cp}$$

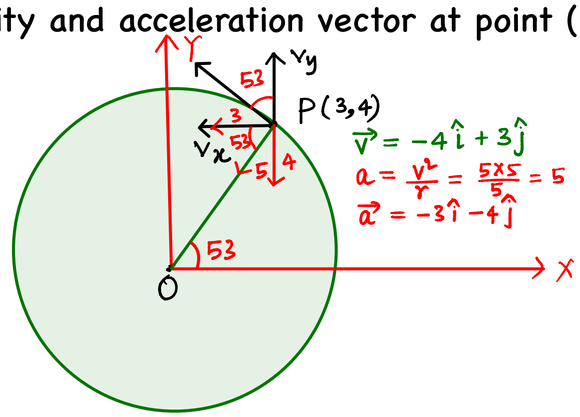
$$\alpha \cancel{r} = \omega^2 \cancel{r}$$

$$4 = 16t^2 \Rightarrow t^2 = \frac{1}{4} \Rightarrow t = \frac{1}{2} \text{ sec}$$

2. A particle moves on a circular path with constant speed of 5 m/s in XY plane . Center of circle is at origin and when it passes through a point (3,4) it's y-component of its velocity is positive then find

(i) radius of circle

(ii) right velocity and acceleration vector at point (3,4)



$$\vec{v} = -4\hat{i} + 3\hat{j}$$

$$a = \frac{v^2}{r} = \frac{5 \times 5}{5} = 5$$

$$\vec{a} = -3\hat{i} - 4\hat{j}$$

$v = 5$ m/s = Const

$a_T = 0$

a_{cp}

3. A particle moves in a circular path of radius 3m at a speed given as v=3t^2 m/s. Find at t=2 sec (A). Tangential accelaration . (B). Normal accelaration (c) total accelaration.

12 48

$$\frac{v^2}{R} = \frac{(3t^2)^2}{R} = \frac{12 \times 12^4}{3} = 48 \qquad 12\sqrt{17}$$

$$\sqrt{48^2 + 12^2} = \sqrt{(12 \cdot 4)^2 + 12^2} = 12\sqrt{17}$$

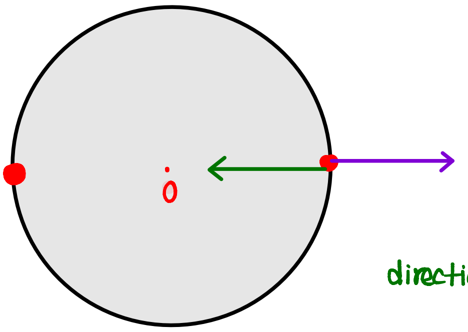
4. A particle is projected at a speed u at an angle θ with the horizontal. Find the radius of curvature at the highest point of the trajectory of projectile.

Centripetal force

Force responsible for centripetal accelaration.
Responsible force for moving the particle in circular path
Magnitude = mass × centripetal accelaration

$$= \frac{mv^2}{R}$$

Centrifugal force

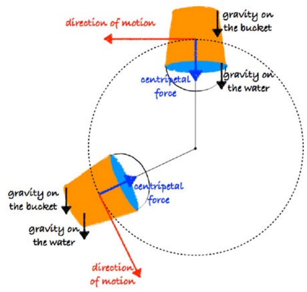


Pseudo Force

$-ma$

$\frac{mv^2}{r}$

direction = away from center.

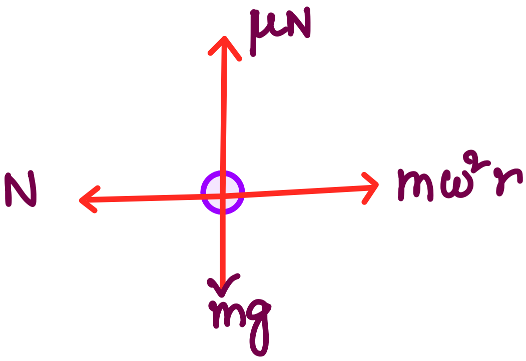


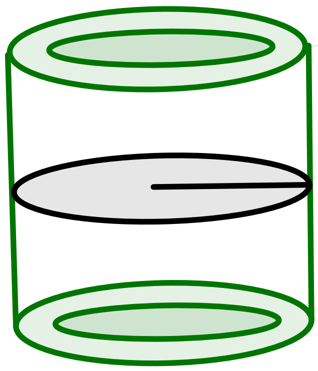
$$\cancel{m}g = \frac{\cancel{m}v^2}{R}$$

$$v^2 = gR$$

$$\boxed{v = \sqrt{gR}}$$

Dynamics of circular motion





$$\cancel{m}g = \mu \cancel{m}\omega^2 r$$

$$\omega^2 = \frac{g}{\mu R}$$

$$\omega = \sqrt{\frac{g}{\mu R}}$$

Minimum angular velocity of drum.

Conical pendulam

$T\cos\theta = mg$ —— (i)

$T\sin\theta = \dfrac{mv^2}{R}$ —— (ii)

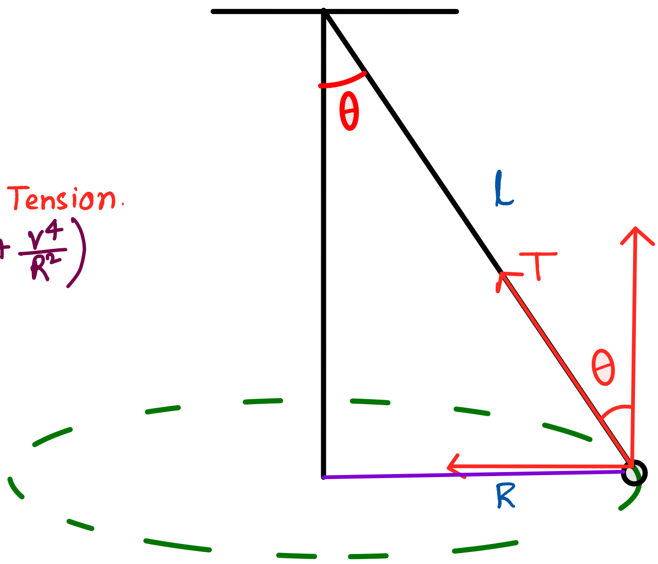
$\tan\theta = \dfrac{v^2}{Rg}$

$R = l\sin\theta$

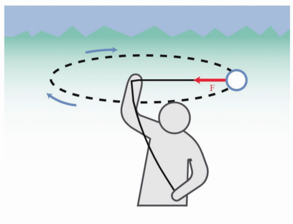
Time period $T = \dfrac{2\pi r}{v}$

$(i)^2 + (ii)^2 =$ Tension.

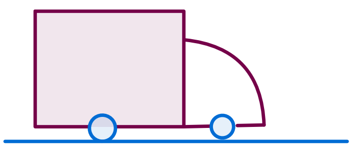
$T^2 = m^2\left(g^2 + \dfrac{v^4}{R^2}\right)$



A rope with a length of 2.0 m is used to swing a 0.25 kg object in a horizontal circular motion with a uniform speed of 20.0 m/s. What is the force of tension in the rope?



Circular turning on flat road by friction only



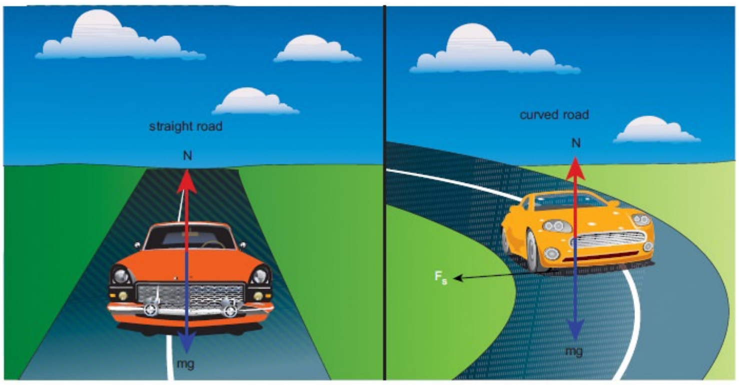
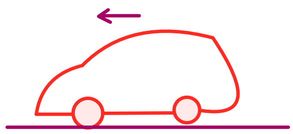


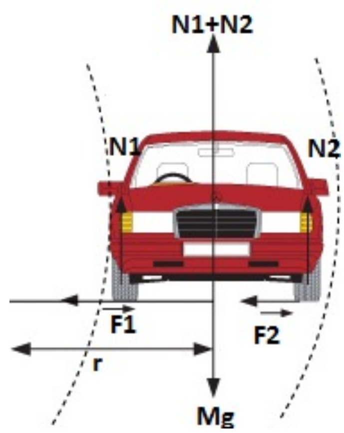
Figure 3.43 Forces acting on the vehicle on a leveled circular road

At limiting case

$\dfrac{\cancel{m}v^2}{R} = \mu \cancel{m} g$

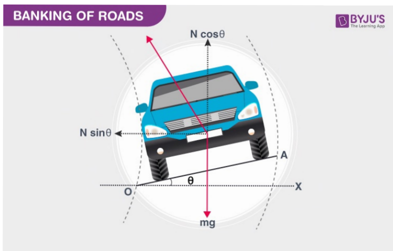
$v^2 = \mu R g$





$V_{max} = \sqrt{\mu r g}$

Circular turning on roads by banking of roads only



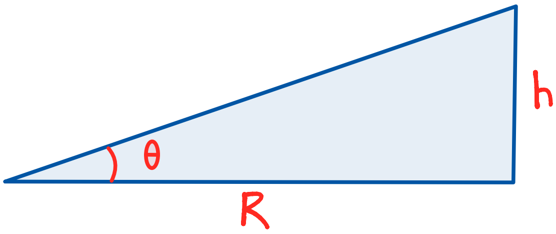
$$N\sin\theta = \frac{mv^2}{R} \quad \text{—— (i)}$$

$$N\cos\theta = mg \quad \text{—— (ii)}$$

$$\tan\theta = \frac{v^2}{Rg} = \sin\theta$$

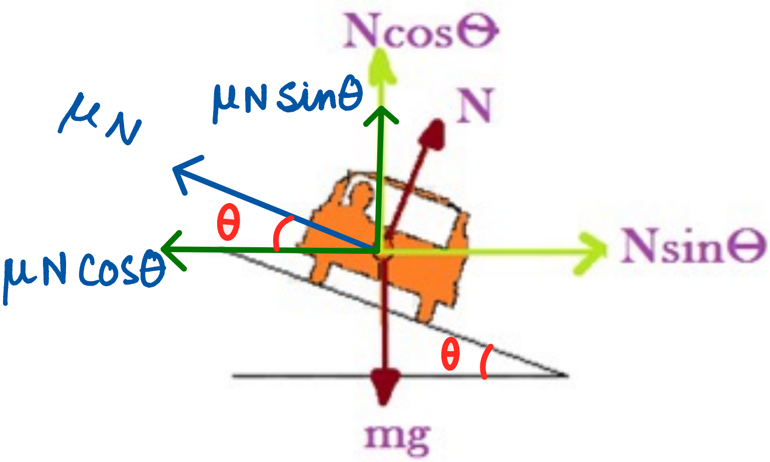
θ very small.

$$\tan\theta = \sin\theta = \theta$$



Circular turning on roads by friction and banking of road both

Minimum speed



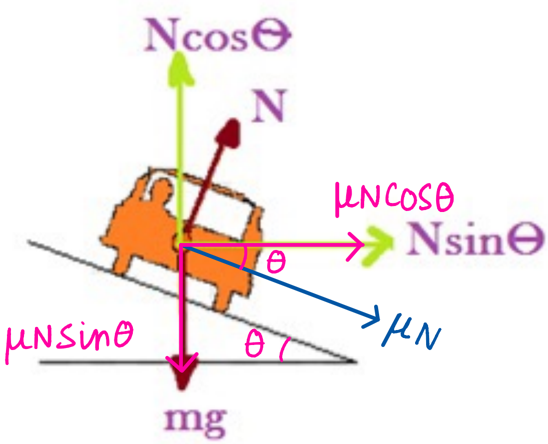
$$\frac{mv^2}{r} = N\sin\theta - \mu N\cos\theta \quad \text{—— (i)}$$

$$mg = N\cos\theta + \mu N\sin\theta \quad \text{—— (ii)}$$

$$\frac{v^2}{rg} = \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}$$

$$V_{min} = \sqrt{rg\left[\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right]}$$

Maximum speed



$$N\cos\theta = mg + \mu N\sin\theta$$

$$N\sin\theta = \frac{mv^2}{R} - \mu N\cos\theta$$

$$V_{max} = \sqrt{rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

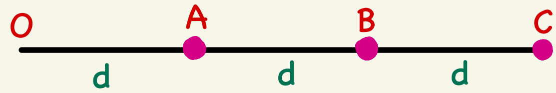
(1). A rod is 8 m wide. Its average radius of curvature is 40 m. The outer edge is even though lower edge by a distance of 1.28 m. Find the velocity of vehicle for which the road is most suited?

(2). Keeping the banking angle of the road constant, the maximum safe speed of passing vehicles is to be increased by 10%. The radius of curvature of the road will have to change from 20 m to ________________

(3). A block of mass 2 KG is tied to a string of length 2m , the other end of which is fixed. The block is moved on a smooth horizontal table with constant speed 5 m/s. Find the tension in the string ?

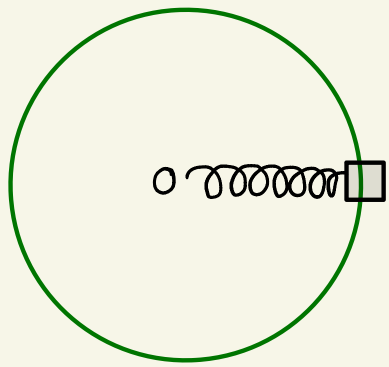
(4). A block of mass 2 KG is tied to a string of length 2m, the other end of which is fixed. The block is moved on a smooth horizontal table with a constant speed, if braking strength of string is 100 N, then what can be the maximum possible speed of particle for circular motion?

(5). Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in horizontal plane. If the velocity of the outermost particle is V_0 , Then the ratio of tensions in the three section of the string is

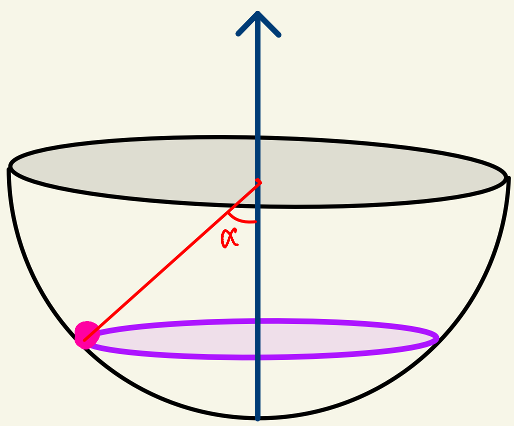


(6). A coin of mass M is kept on the age of horizontal rotating platform of radius are, which is rotating with constant angular velocity u . If coefficient of friction is u, find friction force between coin and platform?

(7). A block of mass M is tied to a spring constant k, natural and small L, and the other end of the spring is fixed O. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity u , then find elongation in sping?



(8). A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without sleeping on its surface. If the surface of bowl is smooth and angle made by radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating?



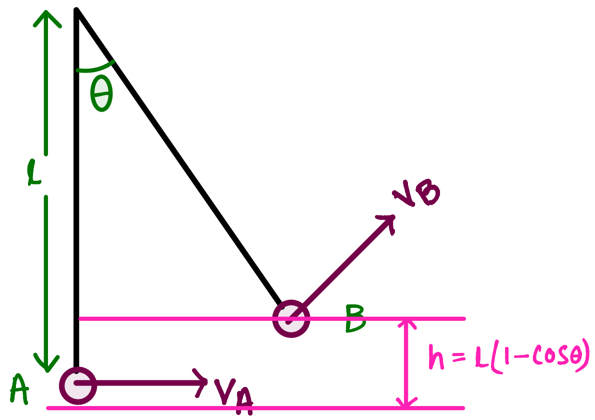
(9). A particle starts moving along a circle of radius $20/\pi$ and with constant tangential acceleration. If velocity of particle is 50 m/s at the end of second revolution after the motion has begun, then find tangential accelaration

(10). The stone is tied to 80 cm long string and executes circular motion with constant speed horizontally. If stone makes 14 revolutions in 25 second, then find the magnitude of net acceleration

(11). A road is 8 m wide. Its average radius of curvature is 40 m. The outer age is above the lower age by a distance of 1.28 m. Find the velocity of vehicle for which route is most suited? ($g = 10\ m/s^2$)

(12). Keeping the banking angle of the road constant, the maximum safe speed of passing vehicles is to be increased by 10%. The radius of curvature of the road will have to change from 20 m to -__________

Vertical circular motion



$$ME_A = ME_B$$

$$K.E_A = KE_B + PE_B - PE_A$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgL(1-\cos\theta)$$

$$v_B^2 = v_A^2 - 2gL(1-\cos\theta)$$

$$v_B = \sqrt{v_A^2 - 2gL(1-\cos\theta)}$$

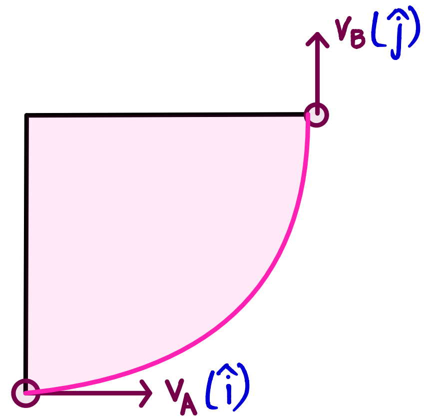
at $\theta = \pi/2$, $v_B = \sqrt{v_A^2 - 2gL(1-0)}$

$$v_B = \sqrt{v_A^2 - 2gL}$$

$$\Delta\vec{v} = \vec{v_f} - \vec{v_i}$$

$$= \sqrt{v_A^2 - 2gL}\ \hat{j} - v_A\hat{i}$$

$$\Delta v = |\Delta\vec{v}| = \sqrt{2v_A^2 - 2gL}$$

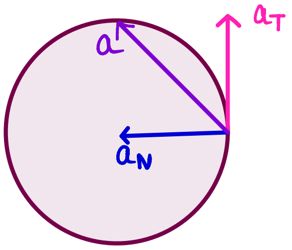


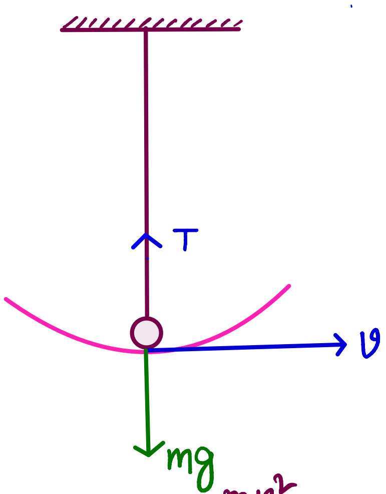
Non uniform circular motion

speed variable.

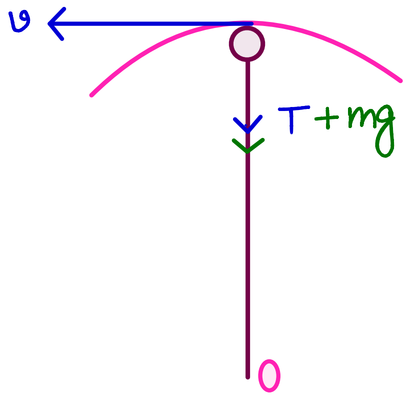
$a_T \neq 0$

$a_T , a_N \qquad a = \sqrt{a_T^2 + a_N^2}$

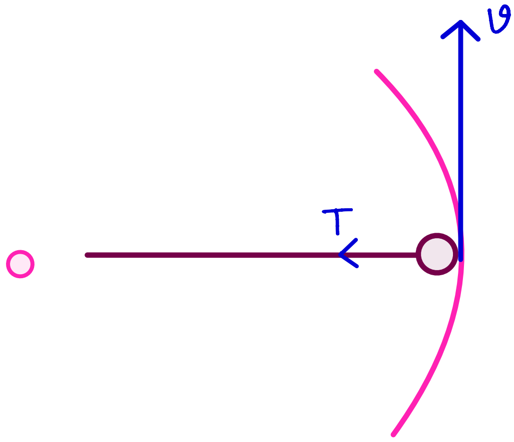




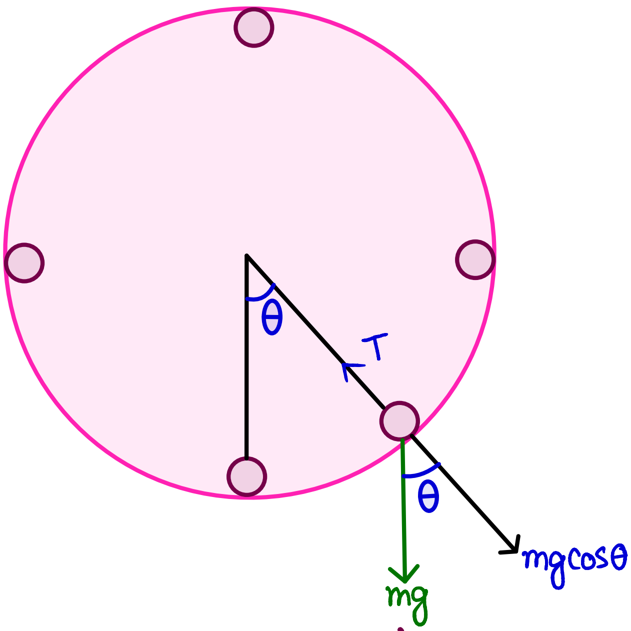
Tension = $T = \frac{mv^2}{R} + mg$



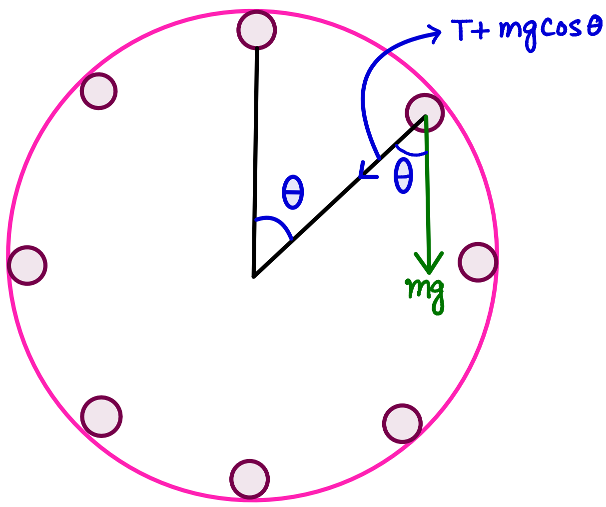
Tension $T = \frac{mv^2}{R} - mg$



Tension $T = \frac{mv^2}{R}$



Tension = $T = \frac{mv^2}{R} + mg\cos\theta$



Tension $T = \frac{mv^2}{R} - mg\cos\theta$

Conclusion

At bottom

$\frac{mv^2}{R} + mg$

At topmost point

$\frac{mv^2}{R} - mg$

At horizontal position

mv^2/R

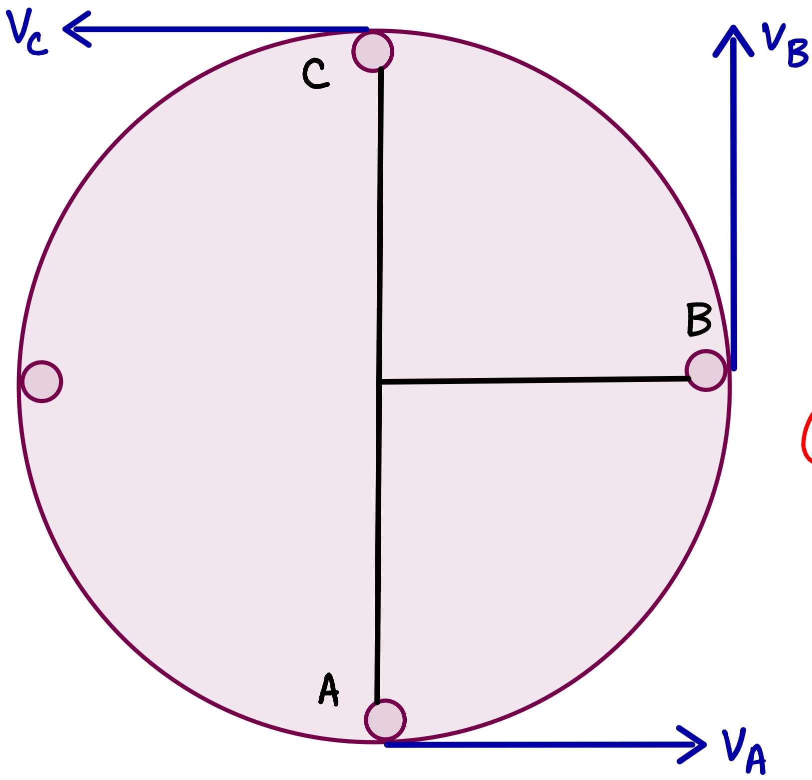
At any arbitrary position ($\theta < 90$)

$\frac{mv^2}{R} + mg\cos\theta$

At any arbitrary position ($\theta > 90$)

$\frac{mv^2}{R} - mg\cos\theta$

Vertical circular motion



Condition for just completing the vertical circle

$$T_c \geq 0$$

$$\frac{mv^2}{L} - mg \geq 0$$

$$v^2 \geq gL$$

Minimum speed at C is $V = \sqrt{gl}$

Minimum speed of projection OR minimum speed at A

Decrease in K.E = increase in P.E

$$\frac{1}{2}\cancel{m}\left(V_A^2 - V_B^2\right) = \cancel{m}g\,(2L)$$

$$V_A^2 = 4gL + V_B^2 = 4gL + gL$$

$$V_A = \sqrt{5gL}$$

Tension at A :

$$T = \frac{mV_A^2}{L} + mg$$

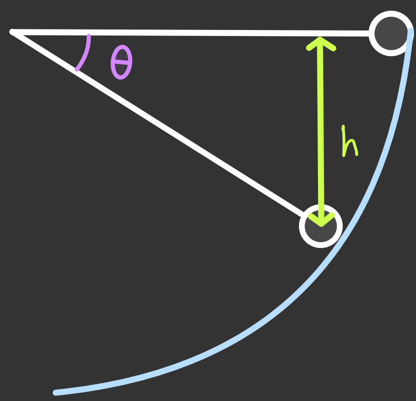
$$= \frac{m}{\cancel{L}} \times 5g\cancel{L} + mg$$

$$T = 6mg$$

Summary

	Bottom.	~~Topmost.~~ Horizontal	~~Horizontal~~ Topmost
Minimum tension at	6mg	3mg	0
Minimum velocity at	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$

Concept-1 : Velocity of bob released from horizontal position

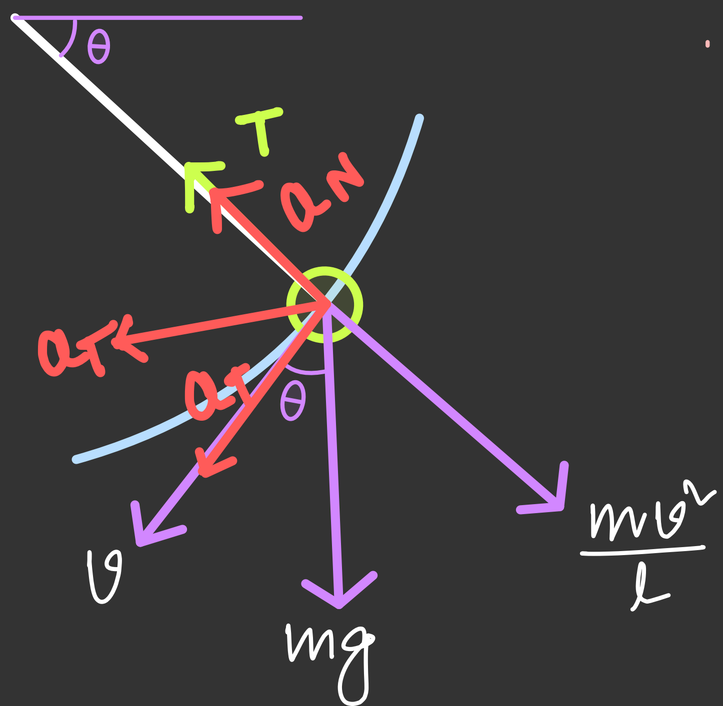


$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2gl\sin\theta}$$

Tension in string when bob is released from horizontal position



$$T = \frac{mv^2}{l} + mg\sin\theta$$

$$T = 3mg\sin\theta$$

Tangential, normal, total acceleration of bob in downward motion of bob released from horizontal position

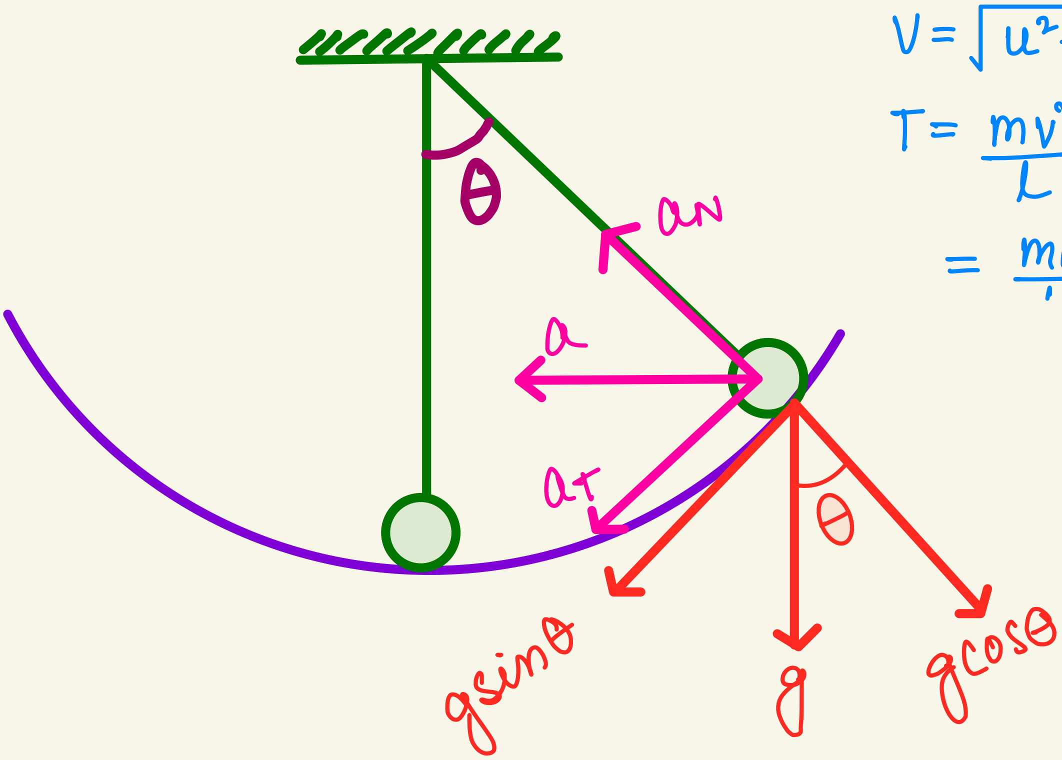
$$a_T = g\cos\theta$$

$$a_N = \frac{v^2}{l} = 2g\sin\theta$$

$$a = \sqrt{a_T^2 + a_N^2}$$

$$= g\sqrt{\cos^2\theta + 4\sin^2\theta}$$

Velocity of bob , tension in string in upward motion starting from bottom point



$$V = \sqrt{u^2 - 2gL(1-\cos\theta)}$$

$$T = \frac{mv^2}{L} + mg\cos\theta$$

$$= \frac{mu^2}{l} - 2mg + mg\cos\theta$$

Tangential, normal , total acceleration of bob in upward motion starting from bottom point

Tangential accelaration $= -g\sin\theta$ (a_T)

normal accelaration $= \dfrac{v^2}{l} = \dfrac{u^2}{l} - 2g(1-\cos\theta)$ (a_N)

Total accelaration $a = \sqrt{a_T^2 + a_N^2}$

Angular amplitude of bob in lower half of vertical circular motion

angular amplitude $\theta = \phi$ $\quad V = 0$

$$0 = u^2 - 2gL(1-\cos\theta)$$

$$\cos\theta = \frac{2gL - u^2}{2gL} \qquad \phi = \cos^{-1}\left(\frac{2gL - u^2}{2gL}\right)$$

Angle of slack where string tension becomes zero in upper half of vertical circular motion,

$$T = \frac{mu^2}{L} - 2mg + 3mg\cos\theta$$

at $\theta = \delta$; $T = 0$

$$\delta = \cos^{-1}\left[\frac{2gL - u^2}{3mg}\right]$$

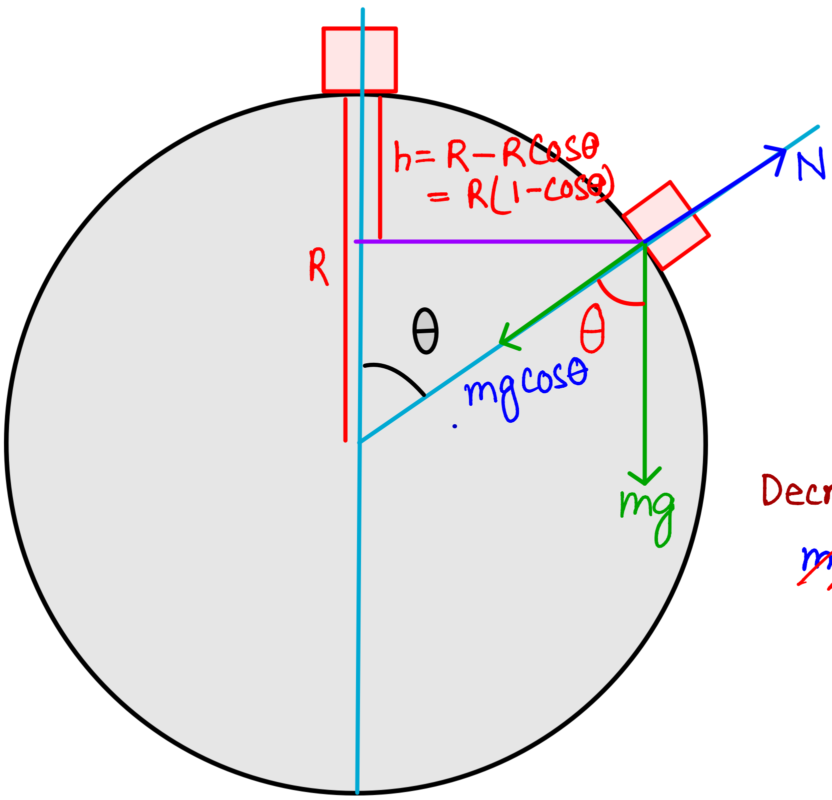
Further this angle particle perform projectile motion.

$$\phi = \cos^{-1}\left(\frac{2gL - u^2}{2gL}\right) \text{ ——— } (1)$$

$$\delta = \cos^{-1}\left(\frac{2gL - u^2}{3gL}\right) \text{ ——— } (2)$$

Case-I : if $u = \sqrt{2gL}$ $\phi = \pi/2$ and $\delta = \pi/2$	Case-II $u > \sqrt{2gL}$ $\delta > \phi$ This is cant possible because maximum angle is ϕ.
Case-III $u = \sqrt{4gL}$ $\delta = \cos^{-1}\left(-\frac{2}{3}\right) = 131^\circ$ $\phi = \pi$.	Case-IV $u = \sqrt{5gL}$ $\phi = \cos^{-1}\left(-\frac{3}{2}\right)$ not possible $\delta = \pi$.

(1). If a body is released from the top of the sphere of radius R,Then find the angle from vertical and height of the body from ground where it leaves the surface bracket (all the surfaces a smooth)



$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R}$$

$$N = 0$$

$$\cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{R}$$

$$v^2 = gR\cos\theta$$

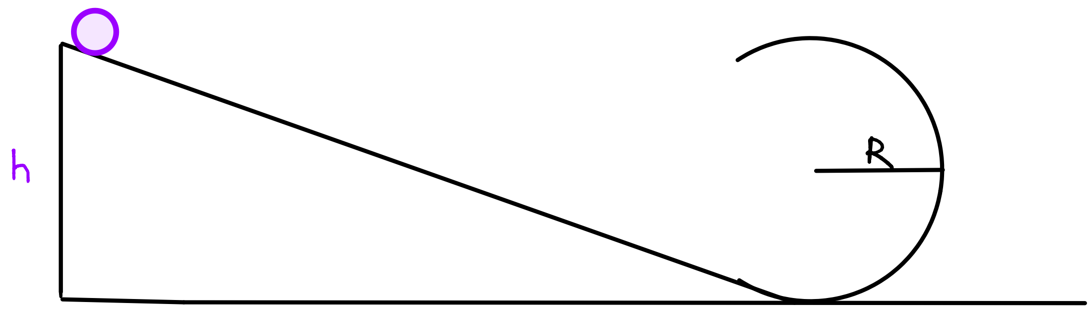
Decrease in P.E = increase in K.E

$$\cancel{mgR}(1-\cos\theta) = \frac{1}{2}\cancel{m}\cancel{gR}\cos\theta$$

$$\Rightarrow \quad 2 - 2\cos\theta = \cos\theta$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

(2).In the given diagram what should be the height h from where our body is released so that it can just complete the vertical circle of radius R.

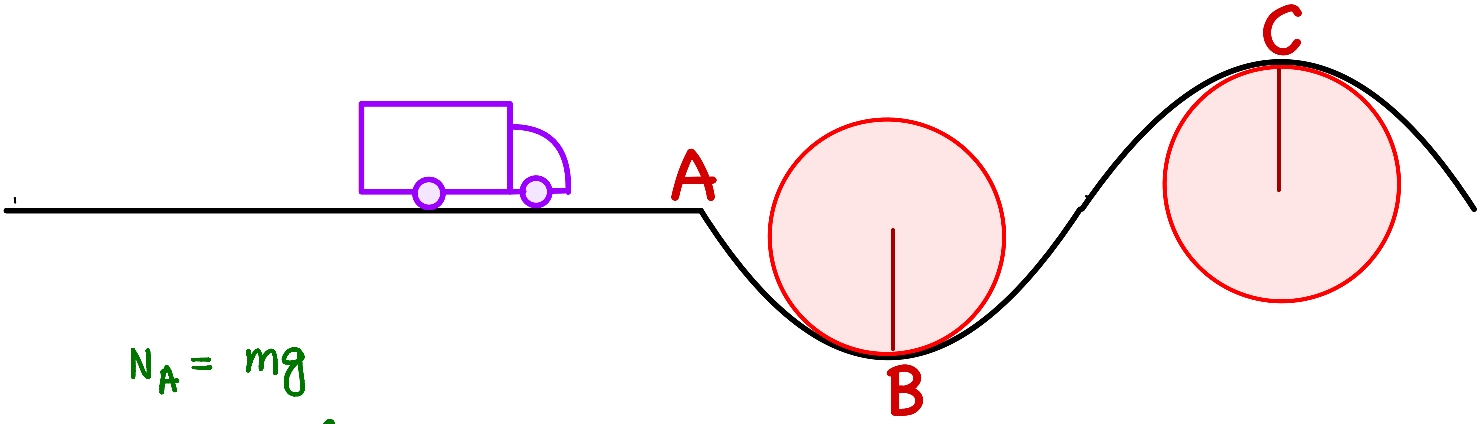


Decrease in P.E = Increase in K.E

$$\cancel{mg}h = \frac{1}{2}\cancel{m}v^2 = \frac{5}{2}\cancel{g}R$$

$$h = \frac{5}{2}R$$

A car is moving along a hilly Road as shown. The coefficient of static friction between the tires and the pavement is constant and the car maintains a steady speed. If at one of the points shown the driver applies break as hard as possible Without making the tyres sleep, the magnitude of the frictional force immediately after the brakes are applied will be maximum if the car was at



$N_A = mg$

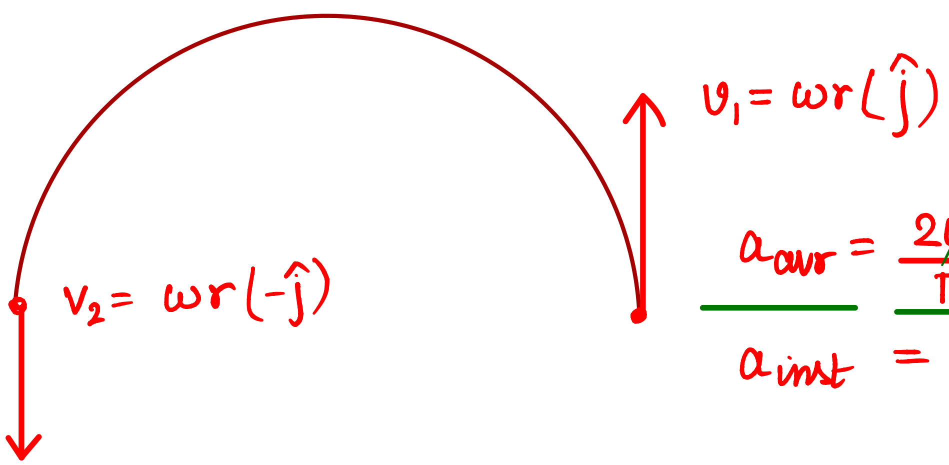
$N_B = \frac{mv^2}{R} + mg$

$N_C = \frac{mv^2}{R} - mg$

$\because \quad N_C < N_A < N_B$

$f_C < f_A < f_B$

For a body in a circular motion with a constant angular velocity , find the ratio of magnitude of the average acceleration over the period of half a revolution to the magnitude of its instantaneous acceleration,

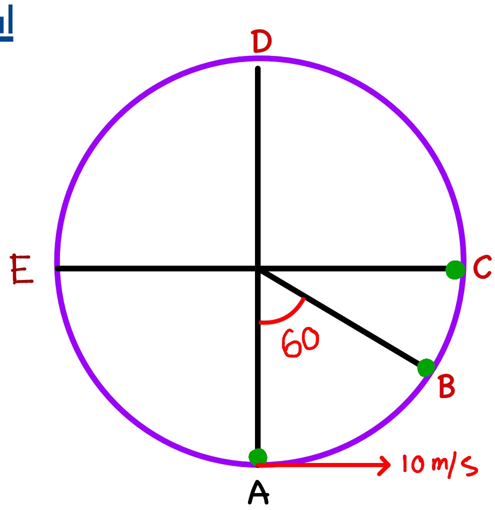


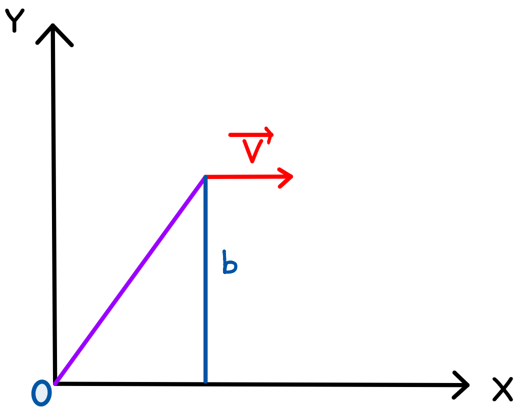
$$\frac{a_{avr} = \frac{2\cancel{\omega}\cancel{r} \times \cancel{\omega}}{\pi}}{a_{inst} = \cancel{\omega^2}\cancel{r}} = \frac{2}{\pi}$$

A wheel has a constant angular acceleration of 3.0 rad /s^2, during a certain 4 sec interval, it turns through an angle of 120 rad. Assuming that at t = 0 ,angular speed equals to 3 rad/s how long is motion at the start of this for second interval

A particle which is connected at one end of a string of length 1 m, passes bottom most point with speed 10 m/s. Find speed at point B, C, D and E

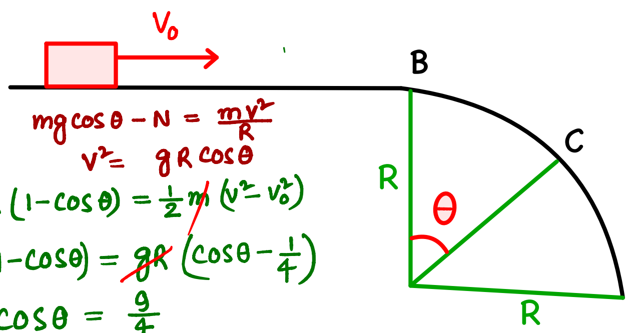
$$V_B = \sqrt{V_A^2 - 2gL(1-\cos\theta)}$$





A particle is moving parallel to X axis as soon in figure such that the Y component of its position vector is constant at all instant and is equal to "b" . Find the angular velocity of particle about the origin when its radius vector makes an angle θ with the X - axis

A Small block slides with a velocity $0.5\sqrt{gr}$ on a horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. Calculate the angle θ shown in the figure



$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$v^2 = gR\cos\theta$$

$$\cancel{m}gR(1-\cos\theta) = \frac{1}{2}\cancel{m}(v^2 - v_0^2)$$

$$2\cancel{gR}(1-\cos\theta) = \cancel{gR}\left(\cos\theta - \frac{1}{4}\right)$$

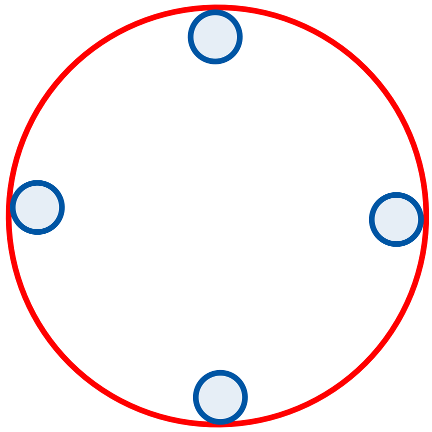
$$\Rightarrow \quad 3\cos\theta = \frac{9}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{3}{4}$$

$$\theta = \cos^{-1}\left(\frac{3}{4}\right)$$

A particle of mass M is moving in a circular path of radius R and is kinetic energy is given by K.E = As^2, where A is positive constant and 's' is distance covered then find the magnitude of net force acting on the particle

A car starts from rest with constant tangential acceleration a in a circular path of radius R. At time T_0 the car skids, find the value of coefficient of friction.



Minimum speed of projection OR minimum speed at A

Decrease in K.E = increase in P.E

$$\frac{1}{2}\cancel{m}(v_A^2 - v_B^2) = \cancel{m}g(2l)$$

$$v_A^2 = 4gl + v_B^2 = 4gl + gl$$

$$v_A = \sqrt{5gl}$$

Tension at A :

$$T = \frac{mv_A^2}{l} + mg$$

$$= \frac{m}{\cancel{l}} \times 5g\cancel{l} + mg$$

$$T = 6mg$$

Summary

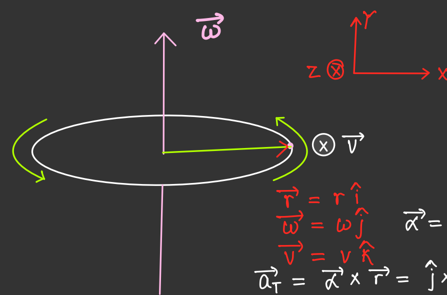
	Bottom.	Horizontal.	Topmost
Minimum value of normal reaction at	6mg	3mg	0
Minimum velocity at	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$

Tangential accelaration

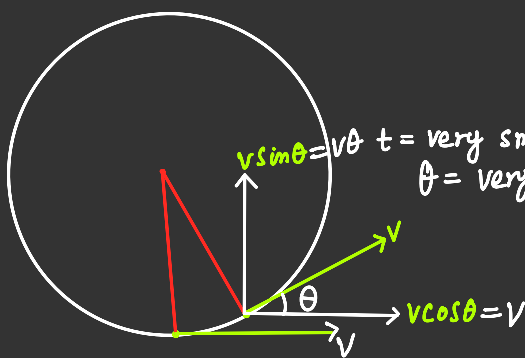
. Acting always along the velocity
Increase the speed.
Acting perpendicular to radius vector.
Acting perpendicular to angular velocity vector.

Vector form $\vec{a}_T = \vec{\alpha} \times \vec{r}$

Magnitude = radius of circular path $\times$ angular acceleration



$\vec{r} = r\hat{i}$
$\vec{\omega} = \omega\hat{j}$ $\vec{\alpha} = \alpha\hat{j}$
$\vec{v} = v\hat{k}$
$\vec{a}_T = \vec{\alpha} \times \vec{r} = \hat{j} \times \hat{i}$



t = very small.
θ = very small
$\cos\theta = 1$
$\sin\theta = \theta$

centripital accelaration $= \dfrac{V\theta}{t}$

$$\omega^2 R = V\omega = \frac{VV}{R} = \frac{V^2}{R}$$

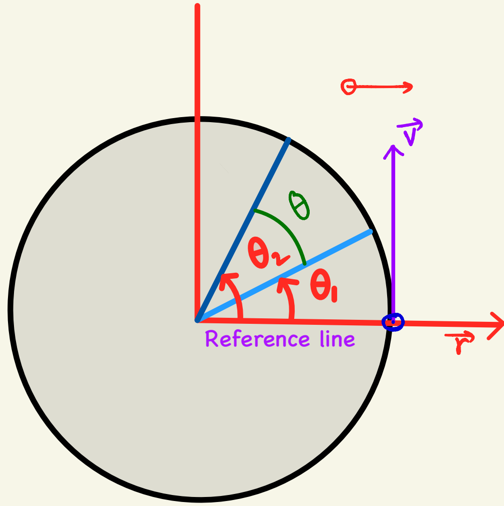
Circular motion

Introduction

If a particle moves in a plane such that its distance from a fixed point remains constant, then motion is called circular motion

The fixed point is known as centre of circle.

The vector joining centre of circle and particle performing circular motion is called radius vector. It has constant magnitude and variable direction



Angular position of circular motion '

Angle made by radius vector from reference line subtended at centre of circle

Angular displacement

Angle through which the position vector of moving particle rotates in a given time interval is called angular displacement.

Axial vector

Unit = radian

$$\text{Angular displacement} = \frac{\text{Arc}}{\text{Radius}}$$

Direction = right hand thumb rule,
rule of cross product

let at t_1 angular position $= \theta_1$

t_2 angular position $= \theta_2$

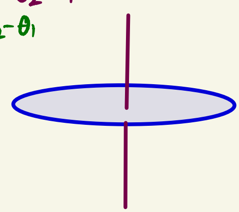
time interval $\Delta t = t_2 - t_1$

change in angular position $= \theta_2 - \theta_1$

$\therefore$ angular displacement $\theta = \theta_2 - \theta_1$

⊙ anticlock = +ve.

⊗ clock = -ve.



Freequency

Number of revelations described by particle per second is its frequency

Unit = hertz, other units are revolution per second, revolutions per minute

1 r.p.s = 60 r.p.m

Time period is time taken by particle to complete one revolution.

$T = 1/n$

Angular velocity, average angular velocity, instantaneous angular velocity

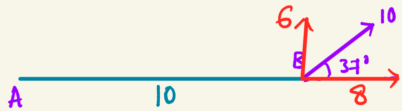
$$\theta = \frac{s}{r} \qquad \frac{\Delta\theta}{\Delta t} \qquad \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$$

$$\frac{s}{t} = \frac{\theta \times r}{t} = \text{const}$$

$$v = r \times \frac{\theta}{t}$$

$$\boxed{v = r \times \omega}$$

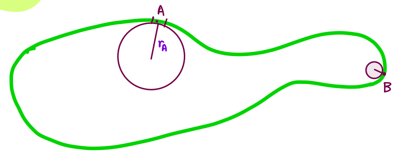
$$\overline{V} = \overline{\omega} \times \overline{r}$$



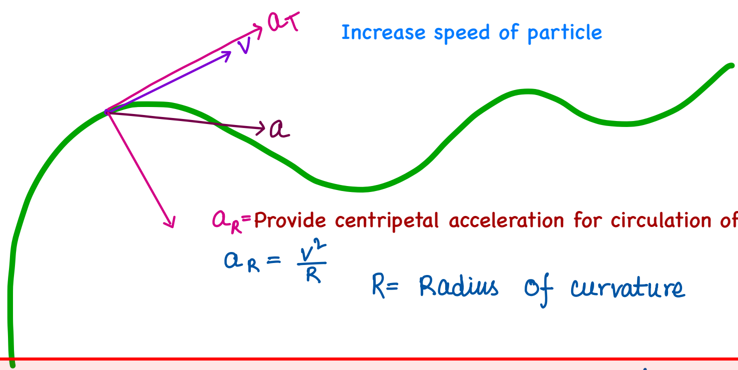
Radius of curvature

Radius of curvature define how much path of the object is curved

For a circular path radius of curvature is constant at all points of the path



Mathematical formula for radius of a curvature $R = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$



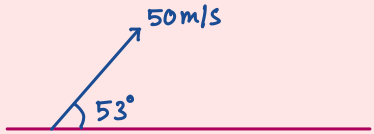
Increase speed of particle

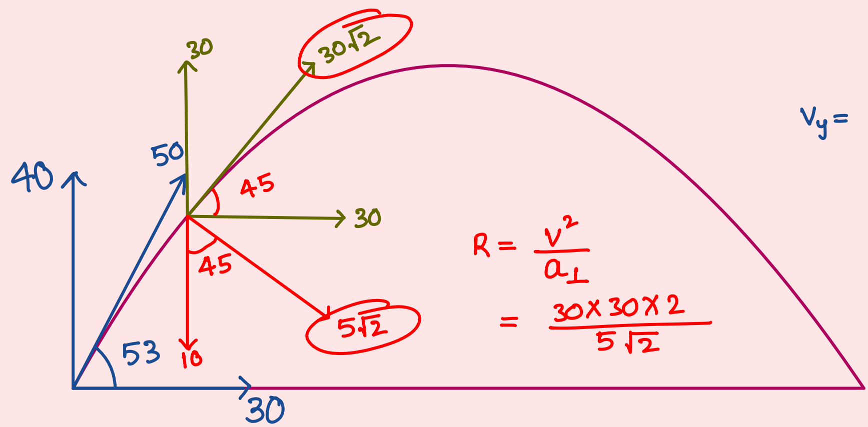
a_R=Provide centripetal acceleration for circulation of the body

$a_R = \dfrac{v^2}{R}$ R= Radius of curvature

Find out radius of curvature after one second

Find radius of curvature at top most point





$V_y = 30 = V_x$

$R = \dfrac{v^2}{a_\perp}$

$= \dfrac{30 \times 30 \times 2}{5\sqrt{2}}$

- $\vec{V} = 3\hat{i} + 4\hat{j}$

  $\vec{a} = \hat{i} + \hat{j}$

Find the radius of curvature

Find tangential acceleration

$a^2 = a_p^2 + a_N^2$

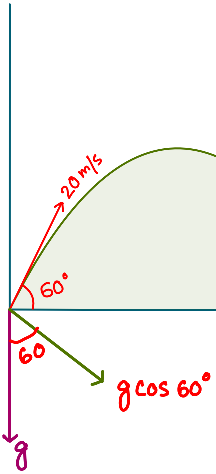
$2 = \left[\dfrac{\vec{a}\cdot\vec{v}}{|\vec{v}|}\right]^2 + a_N^2$

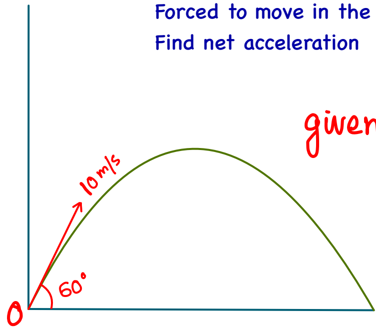
$2 = \dfrac{49}{25} + a_N^2$

$a_N = \dfrac{1}{5}$

$r = \dfrac{5^2}{1/5} = 125$

$a_T = 7/5$





Forced to move in the same trajectory.
Find net acceleration

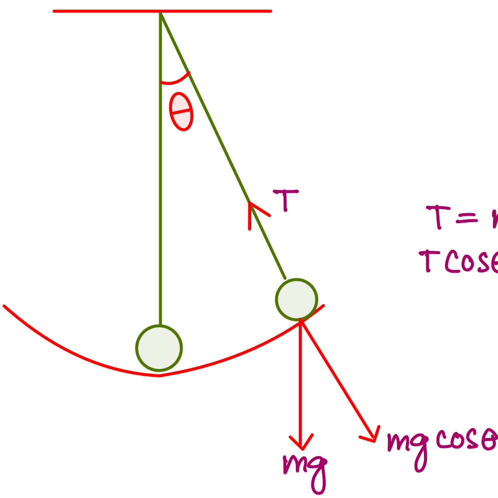
given accelaration at O = 2.5

Radius of curvature $= \dfrac{20^2}{g\cos 60} = \dfrac{10^2}{a_\perp}$

$a_\perp = 0$

$a_{net} = 2.5$

$a_\parallel =$ ____



$T = mg\cos\theta$

$T\cos\theta = mg$

Which one is correct.

Find velocity which bob will have at highest point if it is a given horizontal speed of $\sqrt{4gl}$ at lowermost point. (Length f string is = l)

$$T = \frac{mv^2}{R} - mg\cos\theta$$

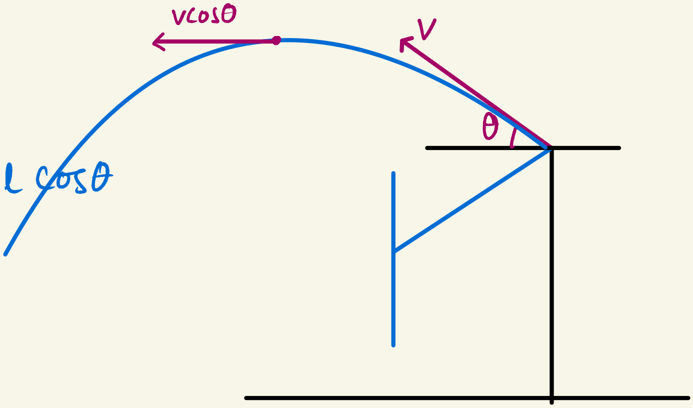
$$v^2 = gR\cos\theta$$

$$v_A^2 - 2gL(1+\cos\theta) = gL\cos\theta$$

$$2gL = 3gL\cos\theta$$

$$\cos\theta = \frac{2}{3}$$

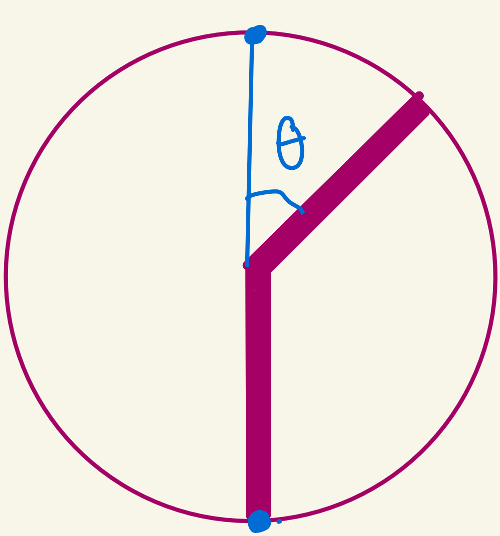
$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$



$$v^2 = \frac{2}{3}gR$$

$$v\cos\theta =$$

A massless rod have a particle of mass m attached to the end of a rod. Find minimum velocity at bottom to complete circular motion.



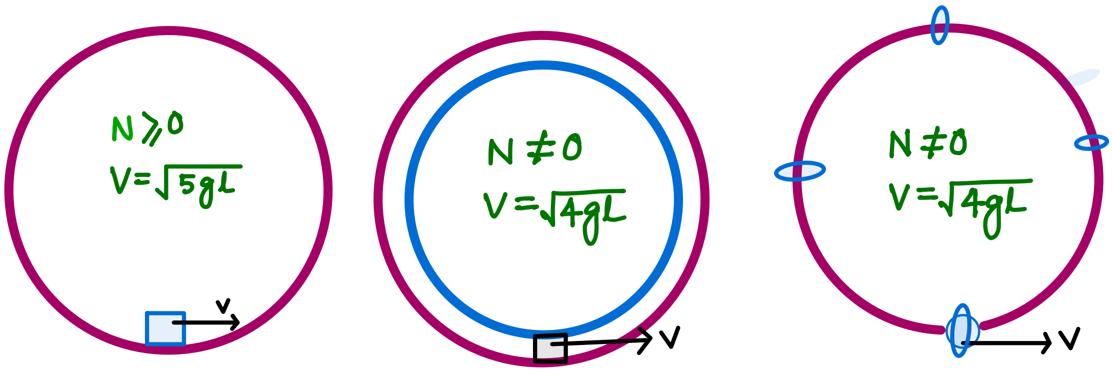
$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g(2R)$$

$$v^2 = 4gR$$

$$v = \sqrt{4gR}$$

NUMERICALS

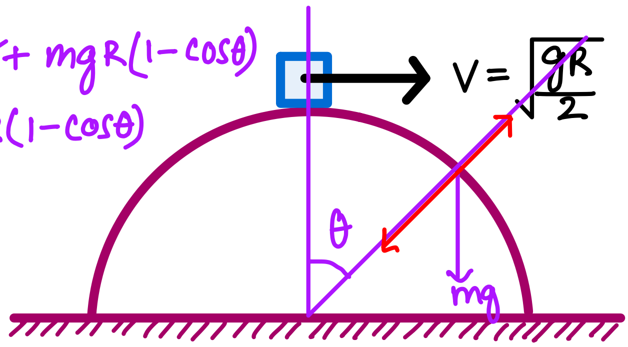
Find minimum velocity of projection for complete circular motion.



Find angle with vertical at which it will loss contact from the surface

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgR(1-\cos\theta)$$

$$v^2 = u^2 + 2gR(1-\cos\theta)$$



$$\frac{mv^2}{R} = mg\cos\theta$$

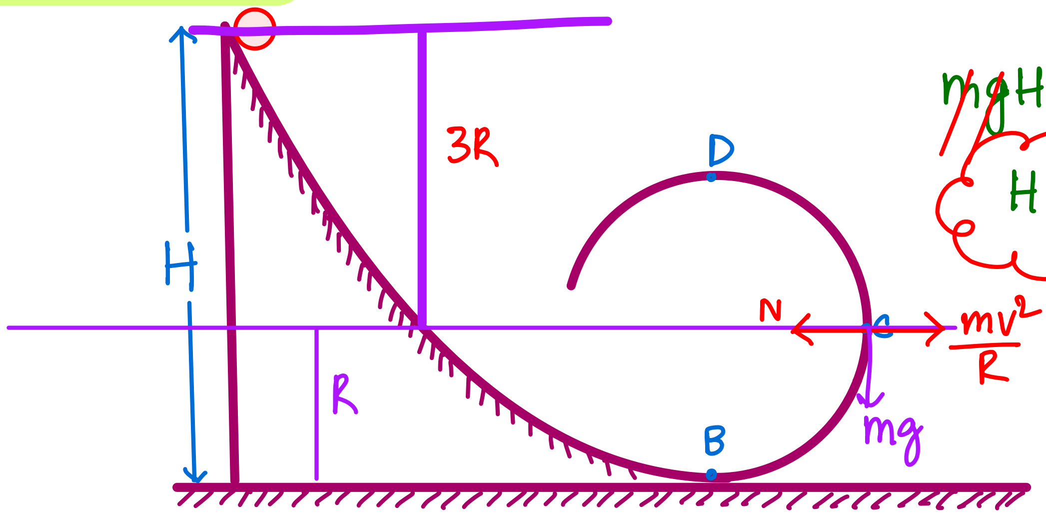
$$\cos\theta = \frac{v^2}{gR}$$

$$\cancel{gR}\cos\theta = \frac{\cancel{gR}}{2} + 2\cancel{gR}(1-\cos\theta)$$

$$6\cos\theta = 5 \Rightarrow \cos\theta = \frac{5}{6}$$

MINIMUM VALUE OF H SO THAT PARTICLE REACH POINT D

IF H=4R, THEN FIND N_C



$$\cancel{m}\cancel{g}H = \frac{1}{2}\cancel{m}\,5\cancel{g}R$$

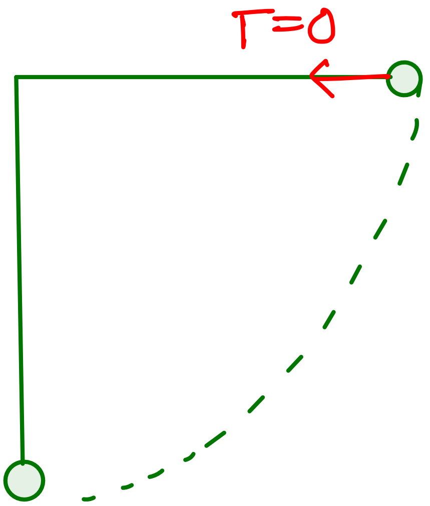
$$H = \frac{5R}{2}$$

$$mg(3R) = \frac{1}{2}mv^2$$

$$\frac{mv^2}{R} = \frac{6mg\cancel{R}}{\cancel{R}} = 6mg$$

NUMERICALS

FIND MINIMUM HORIZONTAL SPEED WHICH BOB IS GIVEN AT LOWER MOST POINT SUCH THAT STRING BECOMES HORIZONTAL (LENGTH OF STRING = L).ALSO CALCULATE TENSION AT LOWERMOST POINT AND HEIGHEST POINT OF TRAJECTORY.



$$mgL = \frac{1}{2}mv^2$$

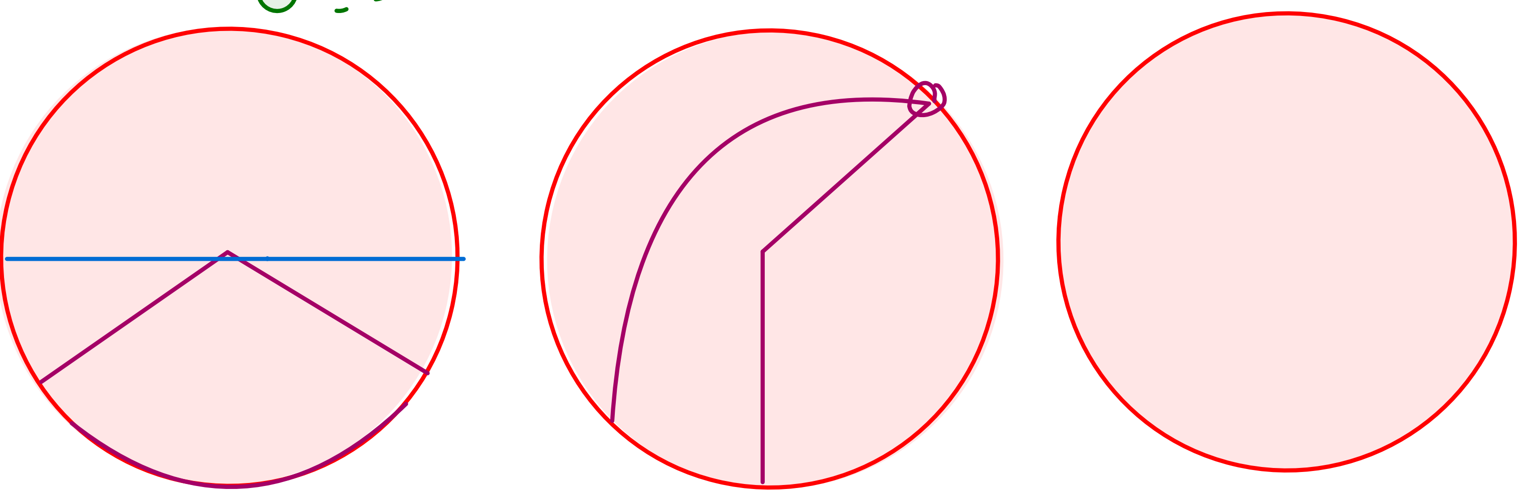
$$v^2 = 2gL$$

$$v = \sqrt{2gL}$$

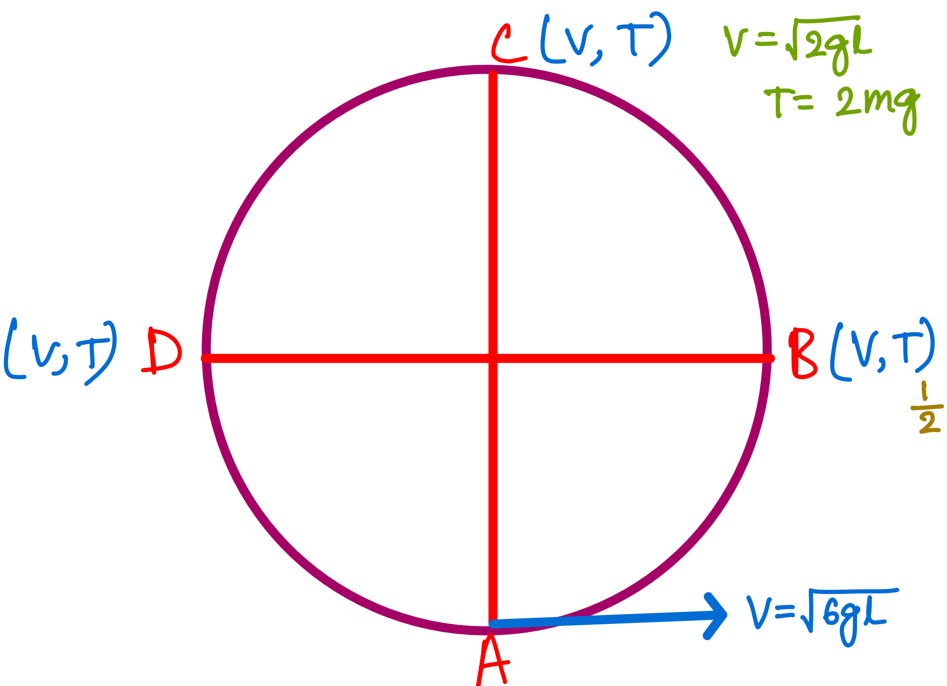
$$T = \frac{mv^2}{L} + mg\cos\theta$$

$T = 0$

$v = \sqrt{gL}$



$v < \sqrt{2gL}$ $\qquad$ $\sqrt{2gL} < v < \sqrt{5gL}$ $\qquad$ $v \geq \sqrt{5gL}$



$v = \sqrt{2gL}$

$T = 2mg$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 = mgL$$

$$u^2 = 6gL - 2gL = 4gL$$

$$T = 4mg$$

Find maximum angle which bob makes with vertical if it is given a horizontal speed $\sqrt{gl}$ at lowermost point. Also calculate tension at lowermost point and highest point of trajectory.

$$T = \frac{mv^2}{R} + mg$$
$$= 2mg$$

$$\frac{1}{2}\cancel{m}\cancel{v^2} = \cancel{m}\cancel{g}\cancel{l}(1 - \cos\theta)$$
$$\theta = 60^\circ$$

$$T = mg\cos\theta$$
$$T = \frac{1}{2}mg$$

$$\boxed{\cancel{g}h = \frac{1}{2} \times 5\cancel{g}R}$$